Algebraic geometry 2

Exercise Sheet 2

PD Dr. Maksim Zhykhovich Summer Semester 2026, 24.04.2026

Exercise 1. Let $\mathcal{F}$ and $\mathcal{G}$ be two sheaves on a topological space X. Show that the presheaf $\mathcal{F} \oplus \mathcal{G}$ is a sheaf (see Exercise 1.24(1) from the lecture).

Exercise 2. 1) Check that the *skyscraper sheaf* defined in Example 1.24(2) is indeed a sheaf.
2) Consider the morphism φ of sheaves on a topological space X constructed in Example 1.24(3). Check that φ is a surjective morphism of sheaves (that is φ_x is surjective on stalks for every $x \in X$) but φ_X is not surjective.

Exercise 3. Let $f : X \to Y$ be a continuous map of topological spaces and let $\mathcal{F}$ be a sheaf on X. Show that the direct image $f_*\mathcal{F}$, $U \mapsto \mathcal{F}(f^{-1}(U))$, is a sheaf on Y.

Exercise 4. Let A be a ring.
(1) Let I, J be two ideals in A. Show that $V(I \cap J) = V(I) \cup V(J)$.
(2) Let $\{I_\alpha\}$ be a family of ideals in A. Show that $V(\sum_\alpha I_\alpha) = \cap_\alpha I_\alpha$.

Exercise 5. Let A be a ring and consider the topological space $\operatorname{Spec} A$ (with Zariski topology). Show that $\operatorname{Spec} A$ is not connected if and only if there exists a non-trivial idempotent $e \in A$ (that is $e^2 = e$ and $e \neq 0, 1$).